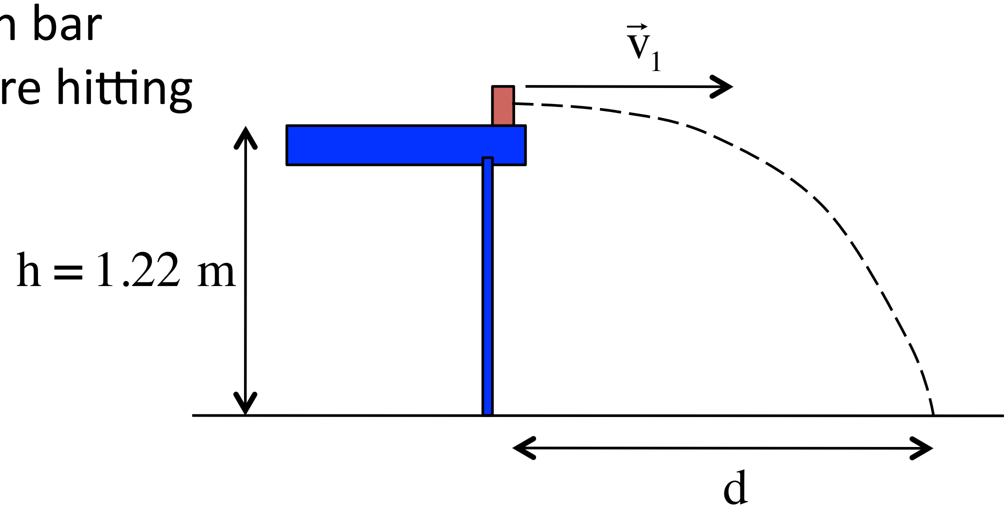
Problem 4.9

A beer mug leaves a 1.22 meter high bar counter and travels 1.4 meters before hitting the floor.

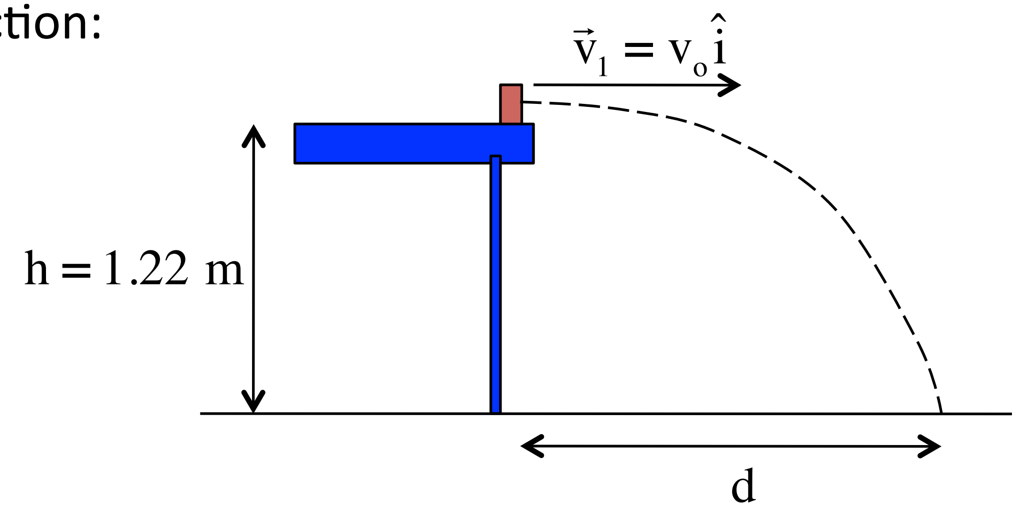


a.) What was it's velocity just before leaving the counter?

Because the acceleration in the x-direction is constant (it's also zero, by the way, assuming no air friction), we can write out kinematic equations associated with motion in that direction. The acceleration is NOT zero in the y-direction, but it is also constant so we can do similarly in *that* direction, too.

Note: I've intentionally laid out the sketch in a reasonable but potentially misleading way (see if you can see where that might have been so). I will point out the potential mess-up as we go, and suggest a way around making that mistake in the write-up.)

Writing out equations in the x-direction:



The only equation that is really useable EVER in the x-direction (assuming you don't have an acceleration-producing jet-pack attached to the mug oriented in the x-direction, or you include air friction), is:

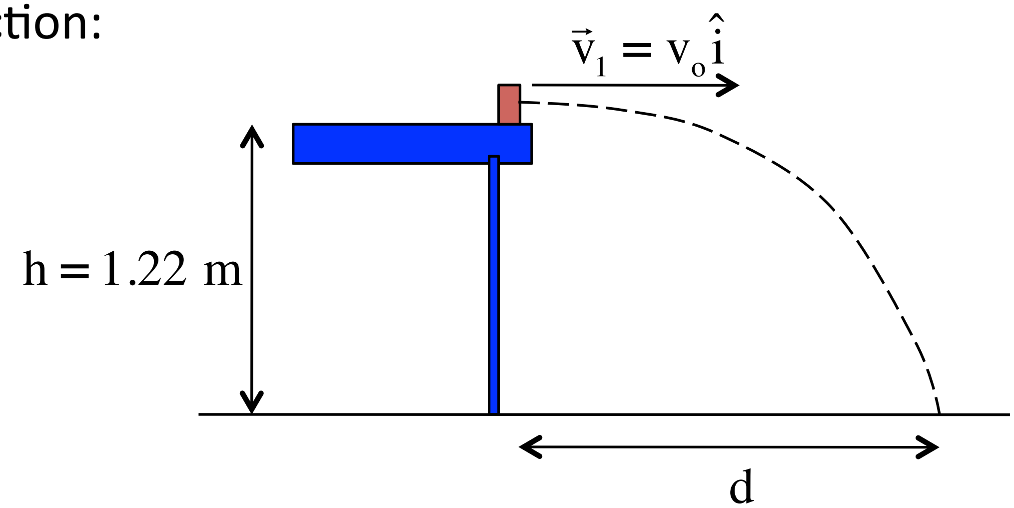
$$\Delta x = v_{1,x}\Delta t + \frac{1}{2}a_x(\Delta t)^2$$

For this problem, that translates into:

$$\Delta x = v_{1,x}\Delta t + \cancel{\frac{1}{2}a_x(\Delta t)^2}^{\,0}$$

$$\Rightarrow \quad d = v_o\Delta t$$

Writing out equations in the y-direction:



In fact, all three kinematic equation are useful in the y-direction, we just have to be clever and choose which works for this problem. In fact, the relationship that fits in this case is

$$\Delta y = v_{1,y}\Delta t + \frac{1}{2}a_y\left(\Delta t\right)^2$$

For this problem, that MIGHT be believed to translate into:

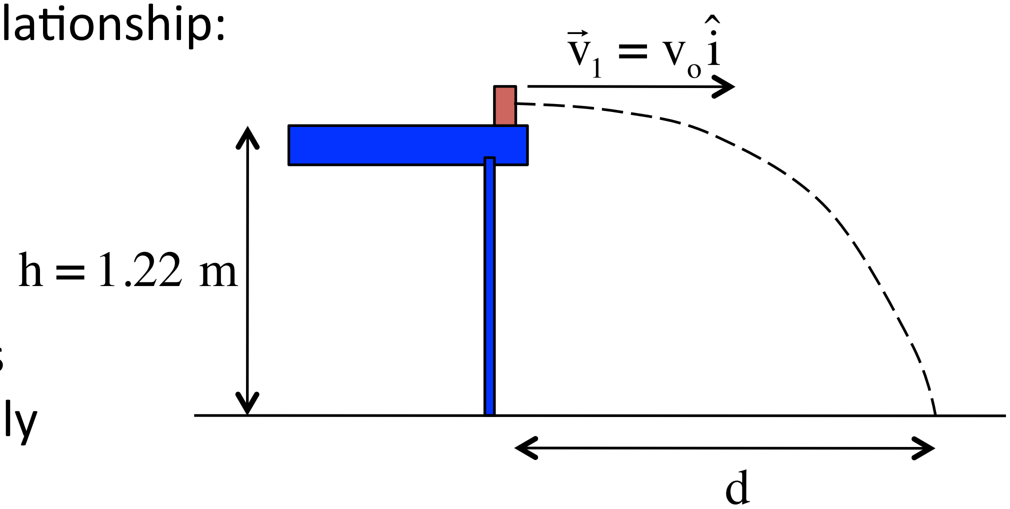
$$\Delta y = \cancelto{0}{v_{1,y}}\Delta t + \frac{1}{2}a_y\left(\Delta t\right)^2$$

$$h = \left(0\right)\Delta t + \frac{1}{2}\left(-g\right)\left(\Delta t\right)^2$$

$$\Rightarrow \quad h = \frac{1}{2}\left(-g\right)\left(\Delta t\right)^2$$

Here is the problem! Look at the relationship:

$$h = \frac{1}{2}(-g)(\Delta t)^2$$



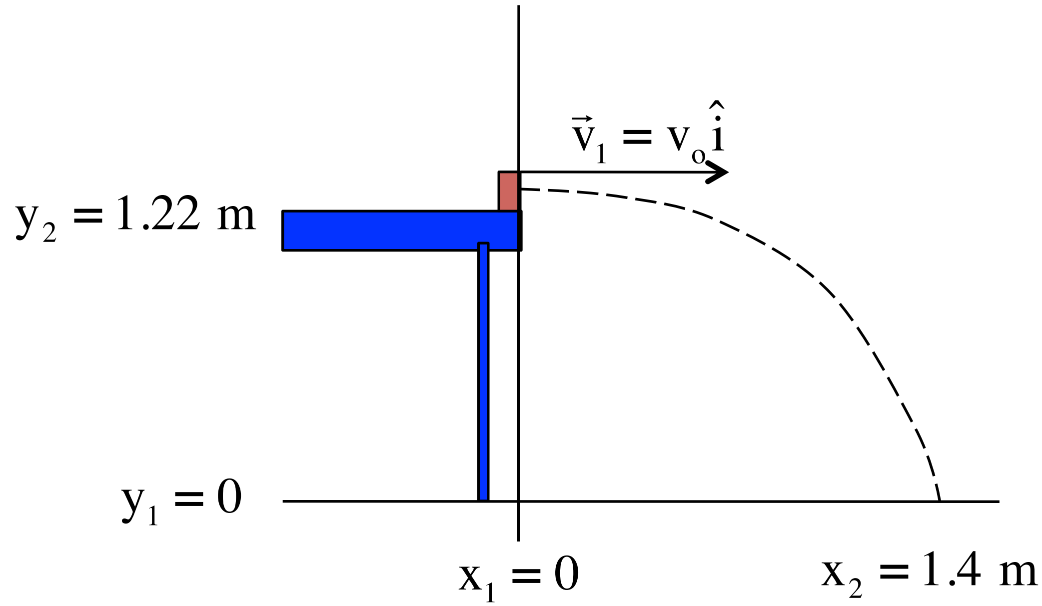
Two things to notice:

First, notice that g = +9.80 m/s/s and, as such, you had to manually place a negative sign into the acceleration relationship.

Second, if we were to solve this relationship as it stands, we'd be asked to take the square root of a *negative number*.

Indeed, you could remedy the problem by making "h" negative (which would be a perfectly OK things to do on an AP test where time limitations are a consideration). When you are not in a timed situation, though, the easiest way to insure you don't inadvertently do something stupid and end up with like this is to identify COORDINATES on your diagram, not distances. (An example of what I'm talking about is found on the next page).

A “better” sketch would have been:



In that way, noting that the final position was at y = 0 and the initial position was at y = 1.22 meters, you could write out the kinematic relationship for the y-motion and end up with:

$$\left(\cancel{y_2}^{\,0} - y_1\right) = \cancel{v_{1,y}}^{\,0}\Delta t + \frac{1}{2}a_y\left(\Delta t\right)^2$$

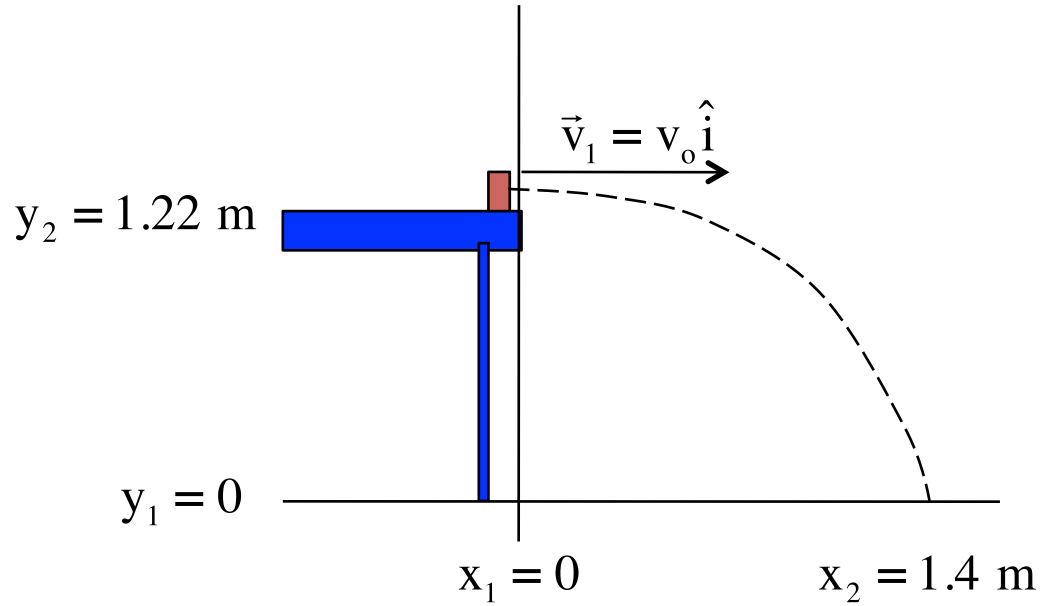
$$\Rightarrow \quad \left(-1.22 \text{ m}\right) = \frac{1}{2}\left(-g\right)\left(\Delta t\right)^2$$

Note that you no longer have the negative problem. The moral of the story is that laying out a sketch with coordinates instead of distances will allow you to forgo divining whether a distance should be positive or negative.

As:

$$\left(-1.22\ \text{m}\right) = \frac{1}{2}\left(-9.8\ \text{m/s}^2\right)\left(\Delta t\right)^2$$

$$\Rightarrow \quad t = .5\ \text{s}$$



we can write:

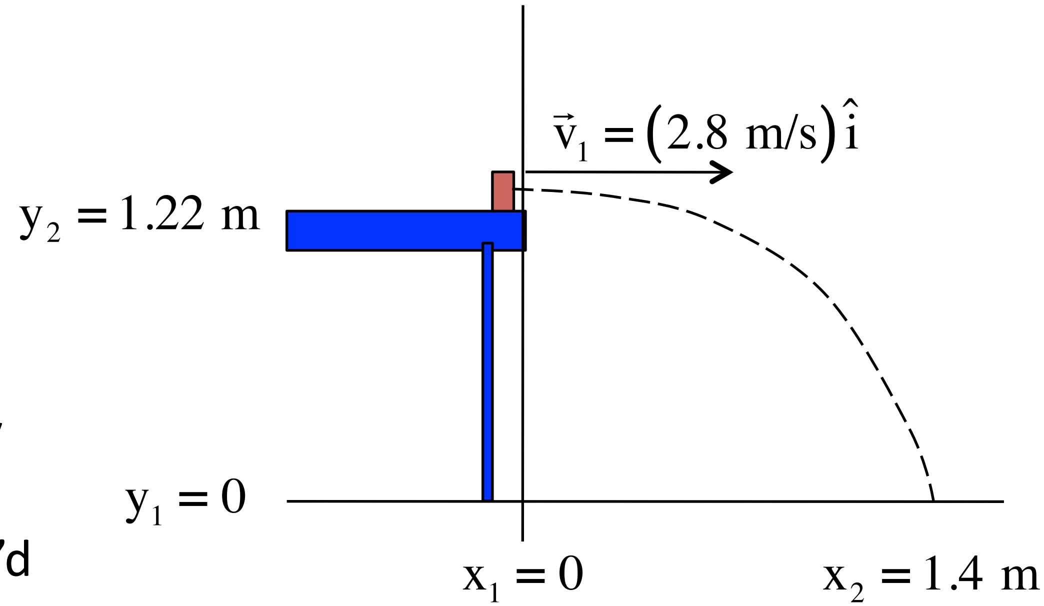
$$x_2 = v_o\Delta t$$

$$\Rightarrow \quad v_o = \frac{1.4\ \text{m}}{.5\ \text{s}}$$

$$= 2.8\ \text{m/s}$$

b.) To determine the angle at which the mug is moving just before it hits the ground, we need both velocity components at that point in time. HALF OF THIS PROBLEM IS ALREADY DONE as the velocity in the x-direction (with its corresponding zero acceleration in that direction) NEVER CHANGES THROUGHOUT THE MOTION. In other words, all we need to calculate is the y-component of the final velocity.

You can get the y component of the "final" velocity two ways. One way:



$$\left(v_{2,y}\right)^2 = \left(v_{1,y}\right)^2 + 2a_y\left(y_2 - y_1\right)$$

(Again, notice I'm writing this with y coordinates, not a delta y. In this case, if you'd messed up the y's you'd again have ended up trying to take the square root of a negative number) Following through:

$$\left(v_{2,y}\right)^2 = \left(\cancelto{0}{v_{2,y}}\right)^2 + 2(-g)\left(\cancelto{0}{y_2} - y_2\right)$$

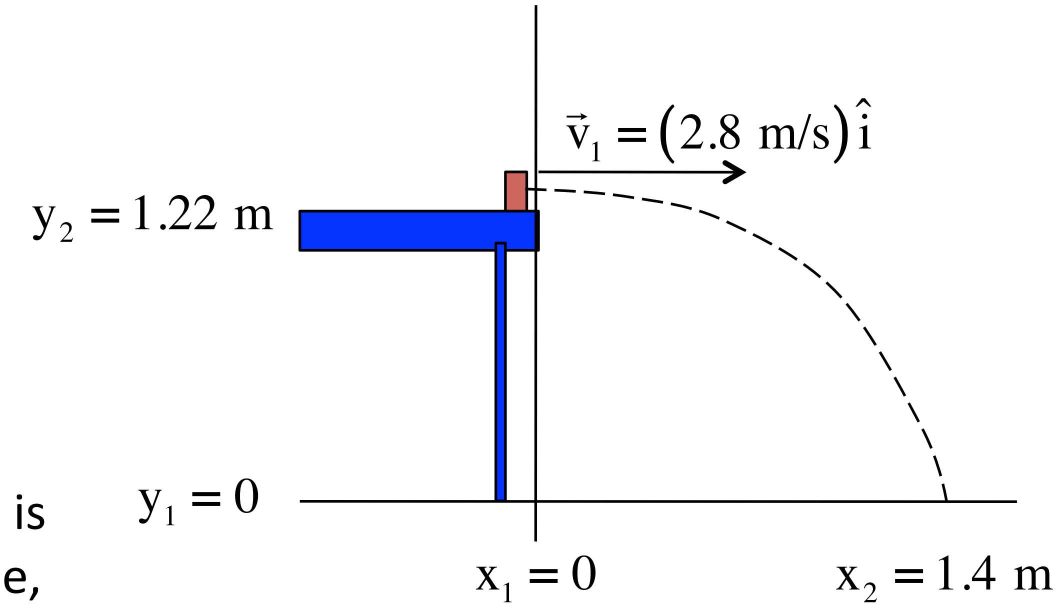
$$\Rightarrow \quad \left(v_{2,y}\right)^2 = 2\left(-9.8\ \text{m/s}^2\right)\left(-1.22\ \text{m}\right)$$

$$\Rightarrow \quad v_{2,y} = 4.89\ \text{m/s}$$

The only problem with this is that with the square root, you have to manually insert the sign of the y-component of the "final" velocity.

The alternate way is:

$$\begin{aligned} v_{2,y} &= \cancelto{0}{v_{1,y}} + a_y \Delta t \\ &= \left(-9.8 \text{ m/s}^2\right)(.5 \text{ s}) \\ &= -4.9 \text{ m/s} \end{aligned}$$



Obviously, the nice thing about this is that the sign is included. In any case, the "final" velocity becomes:

$$\begin{aligned} \vec{v}_2 &= v_{2,x}\hat{i} - v_{2,y}\hat{j} \\ &= (2.8 \text{ m/s})\hat{i} - (4.9 \text{ m/s})\hat{j} \\ &= \sqrt{(2.8 \text{ m/s})^2 - (4.9 \text{ m/s})^2} \measuredangle \tan^{-1}\left(\frac{-4.9}{2.8}\right) \\ &= (5.64 \text{ m/s}) \measuredangle \tan^{-1}(-60.3) \end{aligned}$$